## Framework for Progression in Mathematical Calculation

## Introduction

This document and the progressions for developing effective methods of calculation have been compiled by a consortium of first and middle schools from the Wimborne area and adopted as the agreed route to efficient calculation.

This framework details the key written methods of mathematical calculation to be taught. Its purpose is to promote a consistent and progressive approach to the teaching of mathematical calculation skills, in line with the expectations of the 2014 Mathematics curriculum. Although the main focus of this policy is on the progression to pencil and paper procedures it is important to recognise that the ability to calculate mentally underpins all calculation. Written calculation methods are not a replacement for mental calculation but structures to enable more complex calculations to be carried out efficiently. In every written method there is an element of mental processing. Written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately. Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills, learned in working with a range of manipulative equipment, which provide the foundation for jottings and informal written methods of recording. These mental skills lead on to more formal written methods of calculation. Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When introducing a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept then, as competence increases, larger and more complex numbers can be tackled. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy. The transition between years should not be hurried as not all children will be ready to move on to the next stage at the same time. Progression to the next stage should be made when mastery of the current stage is evident. Progression should be based on attainment across the mathematics curriculum and not just in the calculation processes. Judgements of mastery should be based on evidence gathered in routine classroom assessments and tasks of the pupils competence and accuracy in applying methods learned.

The long-term aim for our children is to have, and be able to select from, a variety of efficient (accurate, reliable and quick) methods of calculation that are appropriate to solve a range of calculation problems.
They should do this by always asking themselves:

- 'Can I do this in my head?'
- 'Can I do this in my head using drawings or jottings?'
- 'Do I need to use a written method?'


## Calculation Framework (NC 2014)

## Addition Mentally (Including Jottings)

| Year | - represent and us |
| :---: | :--- |
| $\mathbf{1}$ | - represent number |
|  | - use number pairs |
|  | - represent doubles |
|  | - represent number |
|  | - use number bond |
|  | - add one-digit and |
|  | - using concrete obja |

- using pictorial representations


Addion Writen
read, write and interpret mathematical statements involving addition (+) and equals (=) signs
$5+7=$

- solve missing number problems (using numbers up to 20 )
$7=\ldots+2$

|  | Subtraction Mentally (Including Jottings) | Subtraction Written |
| :---: | :---: | :---: |
|  | - represent and use subtraction facts within 20, relating them to the corresponding addition facts (see addition) <br> - represent halves of even numbers up to 20 <br> - subtract one-digit and two-digit numbers to 20, including zero <br> - using concrete objects (including exchanging Tens \& Ones) <br> - using pictorial representations | - read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs $15-5=$ <br> - solve missing number problems (using numbers up to 20 ) $7=\_-9$ |
|  | Multiplication Mentally (Including Jottings) | Multiplication Written |
| Year 1 | - calculate the answer to multiplication problems (with the support of the teacher) (using numbers up to 20) <br> - using concrete objects <br> - using pictorial representations <br> - using arrays $5 \times 3=15$ | (mental with jottings only) |


|  | Division Mentally (Including Jottings) |  |  |
| :--- | :--- | :--- | :--- |
| - calculate the answer to division problems (with the support of the teacher) <br> (using numbers up to 20) <br> - using concrete objects | (mental with jottings only) | Division Written |  |

## Calculation Framework (NC 2014)




| Year 2 | Multiplication Mentally (Including Jottings) | Multiplication Written |
| :---: | :---: | :---: |
|  | - recall and use multiplication facts for the 2,5 and 10 multiplication tables <br> - recognise odd and even numbers <br> - calculate mathematical statements for multiplication within the multiplication tables <br> - using materials <br> - using mental methods (with jottings) <br> $5,10,15,20,25$ | - write calculations using the multiplication ( $\times$ ) and equals (=) signs $4 \times 5=20$ <br> - calculate mathematical statements for multiplication within the multiplication tables <br> - using arrays $3 \times 5=15$ <br> - using repeated addition: $\begin{aligned} & 4 \times 5= \\ & 5+5+5+5=20 \end{aligned}$ <br> This can be done on a number line. |
|  | Division Mentally (Including Jottings) | Division Written |
|  | - recall and use division facts for the 2,5 and 10 multiplication tables <br> - use $2 x$ table to divide by 2 , counting in multiples or reversing the calculation <br> - use 10x table to divide by 10, counting in multiples or reversing the calculation <br> - use $5 x$ table to divide by 5 , counting in multiples or reversing the calculation <br> - calculate mathematical statements for division within the multiplication tables - using materials <br> - using mental methods (with jottings) | - write calculations using the division ( $\div$ ) and equals (=) signs $30 \div 2=15$ <br> - calculate mathematical statements for division within the multiplication tables (i.e. no remainders) <br> - using arrays $15 \div 5=3$ <br> - using repeated addition <br> This can be done on a number line. $\begin{aligned} & 15 \div 5= \\ & 510 \quad 15 \end{aligned}$ |

## Calculation Framework (NC 2014)

|  | Addition Mentally (Including Jottings) | Addition Written |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Year } \\ 3 \end{gathered}$ | - add a three-digit number and ones (using numbers up to 1000) <br> - add a three-digit number and tens (using numbers up to 1000) <br> - add a three-digit number and hundreds (using numbers up to 1000) <br> (no bridging the hundreds, bridging the hundreds) | - add numbers with up to three digits, using formal written methods of columnar addition <br> (No exchanging, exchanging ones to tens, exchanging tens to hundreds) |  |  |  |  |
|  | Subtraction Mentally (Including Jottings) <br> - subtract a three-digit number and ones <br> - subtract a three-digit number and tens <br> - subtract a three-digit number and hundreds <br> (no bridging the hundreds, bridging the hundreds) |  |  |  |  |  |
|  |  | - subtract numbers with up to three digits, using formal written methods of columnar subtraction <br> (No exchanging, exchanging tens to ones, exchanging hundreds to tens) |  |  |  |  |

Multiplication Mentally (Including Jottings)

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental strategies

| $35 \times 4$ | $120$ | $35 \times 4$ | $12 \times 10=120$ |
| :---: | :---: | :---: | :---: |
| $70 \times 2=140$ | $35 \times 4$ | $3 \times 4=12$ |  |
|  | 20 | 140 |  |

## Division Mentally (Including Jottings)

- recall and use division facts for the 3, 4 and 8 multiplication tables
$27 \div 3$
$3,6,9,12,15,18,21,24,27$
$9 \times 3=27$
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers progressing to formal written methods
(No exchanging, exchanging ones to tens, exchanging tens to hundreds)

| 13 |
| ---: |
| $\times \quad 38$ |
| 39 | | 24 |
| ---: |

Children should be secure on mental methods of multiplication with jottings before moving on to this standard written method

## Division Written

- write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers (no remainders, remainders)

|  |  |  |  |
| :--- | :--- | :--- | :--- |
| $64 \div 4=16$ | $64 \div 4=16$ | Children should be <br> secure on mental <br> methods of division with <br> $-\frac{40}{24}$ | $10 \times 4$ |
| -24 | $6 \times 4$ | -40 | $10 \times 4$ |
| jottings before moving on |  |  |  |
| to this chunking method |  |  |  |


|  | Addition Mentally (Including Jottings) | Addition Written |
| :---: | :---: | :---: |
| Year 4 | It would be helpful if children could begin to add 4 digit numbers mentally where appropriate, as $\operatorname{Yr} 3$ add 3 digit and $\operatorname{Yr} 5$ add 5 digit. (no bridging, bridging) (include jottings where necessary) $\begin{aligned} & 4 \text { digit }+1 \mathrm{~s} \\ & 4 \text { digit }+10 \mathrm{~s} \\ & 4 \text { digit }+100 \mathrm{~s} \end{aligned}$ | - add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate <br> (Exchanging ones to tens, exchanging tens to hundreds, exchanging hundreds to thousands) <br> Apparatus should be available to support children's standard method calculations |
|  | Subtraction Mentally (Including Jottings) | Subtraction Written |
|  | It would be helpful if children could begin to subtract 4 digit numbers mentally where appropriate, as Yr 3 subtract 3 digit and Yr 5 subtract 5 digit. (no bridging, bridging) (include jottings where necessary) <br> 4 digit - 1s <br> 4 digit - 10s <br> 4 digit - 100s <br> Finding the difference between 2 larger numbers close together can be taught by counting/jumping up from the lowest. $1000-998=2$ | - subtract numbers with up to 4 digits using the formal written methods of columnar and subtraction where appropriate <br> (Exchanging tens to ones, exchanging hundreds to tens, exchanging thousands to hundreds) $\begin{array}{rrr} 31 & 21 & 11 \\ 2345 & 2345 & 2345 \\ -\quad 127 \\ \hline 2218 & -\quad 163 \\ \hline \end{array}$ |
|  | Multiplication Mentally (Including Jottings) | Multiplication Written |
|  | - recall multiplication facts for multiplication tables up to $12 \times 12(6,7,9,11,12)$ <br> - use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1 ; multiplying together three numbers $\begin{aligned} & 35 \times 6 \\ & 3 \times 6=18 \\ & 5 \times 6=30 \\ & 210 \end{aligned}$ | - multiply two-digit and three-digit numbers by a one-digit number using formal written layout |

## Division Mentally (Includeing Jottings)

## Division Written

- recall division facts for multiplication tables up to $12 \times 12$
- write and calculate mathematical statements for division using the multiplication tables that they know including a 3 digit number divided by a 2 digit number. (no remainders, remainders)
- use place value, known and derived facts to divide mentally
$846 \div 6=$
$240 \div 6=$
$240 \div 6=40$
- divide by 1

| 60 | $10 \times 6$ |
| :--- | :--- |
| 60 | $10 \times 6$ |
| 60 | $10 \times 6$ |
| 60 | $10 \times 6$ |

Answer to each subtraction can be added in to the calculation if required. See Yr3.

## Calculation Framework (NC 2014)



## Multiplication Mentally (Including Jottings)

- multiply and divide numbers mentally drawing upon known facts
$45 \times 6=$
$45 \times 2=90,90 \times 3(9 \times 3 \times 10)$
$=270$
$38 \times 15=$
$38 \times 10=380+190(1 / 2$ of 380$)$

$$
=570
$$

- solve problems involving multiplication where larger numbers are used by decomposing them into their factors
$350 \times 25$
$=7 \times 5 \times 10 \times 5 \times 5$
- multiply whole numbers and those involving decimals by 10,100 and 1000
$234.7 \times 10$ becomes
2347.0
$9067.43 \times 100$ becomes 906743.0

When multiplying by 10 and multiples of 10, the decimal point remains fixed and the number moves 1 place to the left for each multiple of 10

## Division Mentally (Including Jottings)

- divide numbers mentally drawing upon known facts
- multiply and divide whole numbers involving decimals by 10, 100 and 1000


## $234.7 \div 10$ becomes

23.47
$9067.43 \div 100$
becomes
 the decimal point remains fixed and the number moves 1 place to the right for each multiple of 10

## - Multiplication Written

- multiply numbers up to 4 digits by a one-digit number using an efficient written method

$$
\begin{array}{r}
4346 \\
\times \quad 8 \\
\hline 34768 \\
234
\end{array}
$$

- multiply numbers up to 4 digits by a two-digit number using long multiplication $4115 \times 25$
4115
125
$\times 2$
20575
82300 (x2, x10)


## 102875

recognise and use square numbers and cube numbers, and the notation
for squared ( ${ }^{2}$ ) and cubed ( ${ }^{3}$ )
$2^{2}=2 \times 2=4$
$3^{3}=3 \times 3 \times 3=27$

- divide numbers up to 4 digits by a one-digit number using the efficient written method of short division and interpret remainders appropriately for the context

$$
72 \div 5
$$

$1368 \div 9$

14 r 2
$5 \longdiv { 2 2 }$
72


## Calculation Framework (NC 2014)



## Division Mentally (Including Jottings)

- perform mental calculations, including with mixed operations and large numbers


## Division Written

- divide numbers up to 4 digits by a two-digit whole number using the efficient written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

| $2666 \div 42$ |  |
| :---: | :---: |
|  | 42) $\longdiv { 2 6 6 6 }$ |
|  | 252 |
|  | 146 |
|  | 126 |
|  | 20 |
| Fraction | 63 20/42 |
|  | 42)2666 |
|  | $\underline{252}$ |
|  | 146 |
|  | 126 |
|  | 20 |
| Decimal | 63.47 |
|  | 42) 266600 |
|  | -252 |
|  | 146 |
|  | -126 |
|  | 200 |
|  | -168 |
|  | 320 |
|  | -294 |
|  | 26 |

$$
\begin{aligned}
& 10 \times 42=420 \\
& 2 \times 42=84 \\
& 4 \times 42=168 \\
& 5 \times 42=210 \\
& 6 \times 42=252
\end{aligned}
$$

Use doubling/halving to find other multiples.

See Appendix 1 for Bar and Double Number Line representations

## Tools to aid multiplication in division calculations

## Bar Method

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 42 | 84$(2 \times 42)$ | 126 | 168 | 210 | 252 | 294 | 336 | 378 | 420 |
|  |  |  | $(42+84)$ | (2x84) | (420/2) | ( $2 \times 126$ ) | (126+268) | ( $2 \times 168$ ) | (336+42) | (10x42) |

## Double Number Line Method

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 42 | 84 | 126 | 168 | 210 | 252 | 294 | 336 | 378 | 420 |
|  |  | ( $2 \times 42$ ) | $(42+84)$ | (2x84) | (420/2) | (2x126) | (126+268) | (2x168) | (336+42) | (10x42) |

This method of multiplication uses know or easily computable facts, by doubling and halving or addition, to construct the data needed to carry out the calculation.
There is no need to calculate all values from 1 times to 10 times, only those needed.

