## Wimborne Area Schools

# Framework for Progression in Mathematical Calculation

St John's First School, Merley First School, , Pamphill First School, Wimborne First School, Witchampton First School, Allenbourn Middle School With thanks to: Henbury View First School and Sturminster Marshall First School

## Framework for Progression in Mathematical Calculation

#### Introduction

This document and the progressions for developing effective methods of calculation have been compiled by a consortium of first and middle schools from the Wimborne area and adopted as the agreed route to efficient calculation.

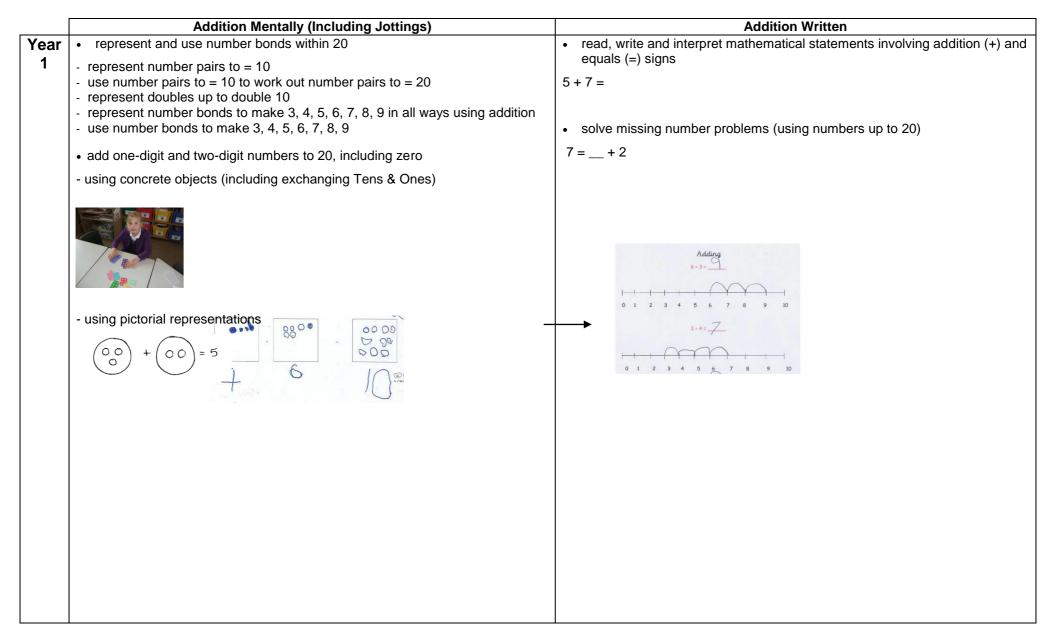
This framework details the key written methods of mathematical calculation to be taught. Its purpose is to promote a consistent and progressive approach to the teaching of mathematical calculation skills, in line with the expectations of the 2014 Mathematics curriculum. Although the main focus of this policy is on the progression to pencil and paper procedures it is important to recognise that the ability to calculate mentally underpins all calculation. Written calculation methods are not a replacement for mental calculation but structures to enable more complex calculations to be carried out efficiently. In every written method there is an element of mental processing. Written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

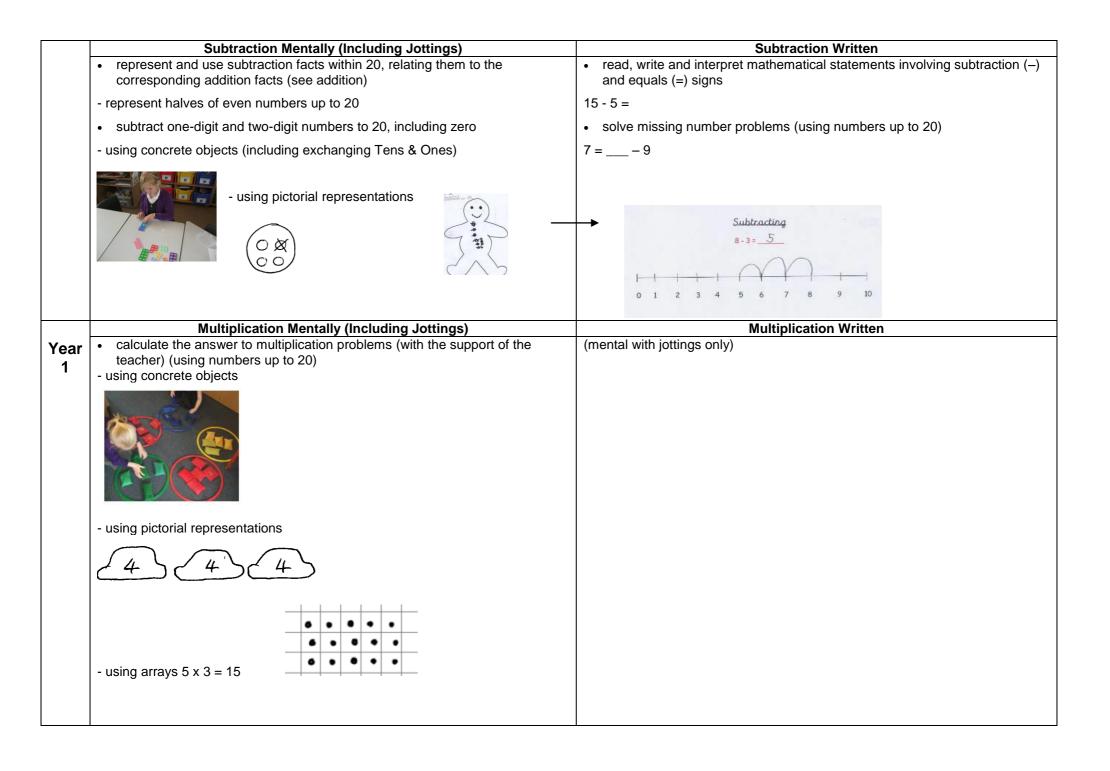
A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately. Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills, learned in working with a range of manipulative equipment, which provide the foundation for jottings and informal written methods of recording. These mental skills lead on to more formal written methods of calculation. Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When introducing a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept then, as competence increases, larger and more complex numbers can be tackled. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy. The transition between years should not be hurried as not all children will be ready to move on to the next stage at the same time. Progression to the next stage should be made when mastery of the current stage is evident. Progression should be based on attainment across the mathematics curriculum and not just in the calculation processes. Judgements of mastery should be based on evidence gathered in routine classroom assessments and tasks of the pupils competence and accuracy in applying methods learned.

The long-term aim for our children is to have, and be able to select from, a variety of efficient (accurate, reliable and quick) methods of calculation that are appropriate to solve a range of calculation problems.

They should do this by always asking themselves:

- 'Can I do this in my head?'
- 'Can I do this in my head using drawings or jottings?'
- 'Do I need to use a written method?'





Division Mentally (Including Jottings)	Division Written
calculate the answer to division problems (with the support of the teacher)     (using numbers up to 20)     using concrete objects	(mental with jottings only)
- using pictorial representations	
- using arrays $15 \div 3 = 5$	

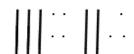
## Addition Mentally (Including Jottings)

## Year 2

- recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- recall number pairs to = 10
- recall number pairs to = 20
- derive number pairs to = 100 in multiples of 10
- derive number pairs to = 100 e.g. 43 + ? = 100
- recall doubles up to double 10
- recall number bonds to make 3, 4, 5, 6, 7, 8, 9 in all ways using addition
- derive additions of multiples of 10
- add a two-digit number and ones (using numbers up to 100)
- add a two-digit number and tens (using numbers up to 100)
- add two two-digit numbers (using numbers up to 100)
- add three one-digit numbers
- using concrete objects (including exchanging Tens & Ones)

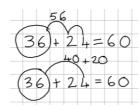


- using pictorial representations



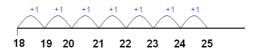
- mentally (with jottings)

			3				
2	50 4	> +	3	ઠે	_	6	Ō



#### **Addition Written**

• add a two-digit number and ones (using numbers up to 100) (no bridging the tens, bridging the tens)



18

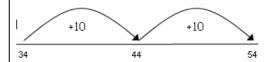
+ 7 25 Calculation to be set out in columns to support the understanding of place value.

However, this should be solved using a range of strategies, not standard column method.

e.g. Number line, Dienes, Numicon, diagrams and jottings. Partitioning & exchanging should be included.

• add a two-digit number and tens (using numbers up to 100)

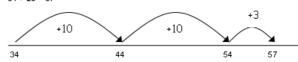
$$34 + 20 =$$



34 + <u>20</u>

NOTE: See text box above.

• add two two-digit numbers (using numbers up to 100) (no bridging the tens, bridging the tens)



NOTE: See text box above.

Year 2

## **Subtraction Mentally (Including Jottings)**

- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- use number bonds to make 3, 4, 5, 6, 7, 8, 9 to work out subtractions i.e. use reversals.
- represent 5 + 3 = 8, therefore 8 3 = 5
- recall halves of even numbers up to 20
- · subtract a two-digit number and ones
- subtract a two-digit number and tens
- subtract two two-digit numbers
- subtract three one-digit numbers
- using concrete objects (including exchanging Tens & Units)



- using pictorial representations

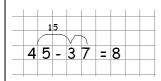


- mentally (with jottings)

$$54 - 20 = 34$$

$$-4 = 30$$

$$-3 = 27$$



#### **Subtraction Written**

• subtract a two-digit number and ones (no bridging the tens, bridging the tens)

15-7=



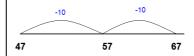
15 <u>- 7</u> Calculation to be set out in columns to support the understanding of place value.

However, this should be solved using a range of strategies, not standard column method.

e.g. Number line, Dienes, Numicon, diagrams and jottings. Partitioning & exchanging should be included.

• subtract a two-digit number and tens

$$67 - 20 =$$



NOTE: See text box above.

• subtract two two-digit numbers (no bridging the tens, bridging the tens)

$$23 - 12 =$$



NOTE: See text box above.

## **Multiplication Mentally (Including Jottings)**

## **Multiplication Written**

Year 2

- recall and use multiplication facts for the 2, 5 and 10 multiplication tables
- · recognise odd and even numbers
- calculate mathematical statements for multiplication within the multiplication tables
- using materials



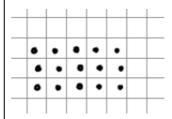
- using mental methods (with jottings)

5, 10, 15, 20, 25



- write calculations using the multiplication (x) and equals (=) signs
- $4 \times 5 = 20$
- calculate mathematical statements for multiplication within the multiplication tables
- using arrays

$$3 \times 5 = 15$$



- using repeated addition:

$$4 \times 5 =$$

$$5 + 5 + 5 + 5 = 20$$

This can be done on a number line.

## **Division Mentally (Including Jottings)**

- recall and use division facts for the 2, 5 and 10 multiplication tables
- use 2x table to divide by 2, counting in multiples or reversing the calculation
- use 10x table to divide by 10, counting in multiples or reversing the calculation
- use 5x table to divide by 5, counting in multiples or reversing the calculation
- · calculate mathematical statements for division within the multiplication tables
- using materials





- using mental methods (with jottings)



write calculations using the division (÷) and equals (=) signs

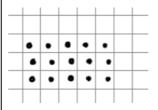
$$30 \div 2 = 15$$

• calculate mathematical statements for division within the multiplication tables (i.e. no remainders)

**Division Written** 

- using arrays

$$15 \div 5 = 3$$



- using repeated addition

5 10 15

This can be done on a number line.

	Addition Mentally (Including Jottings)	Addition Written						
Year 3	<ul> <li>add a three-digit number and ones (using numbers up to 1000)</li> <li>add a three-digit number and tens (using numbers up to 1000)</li> </ul>	add numbers with up to three digits, using formal written methods of columnar addition  (No exchanging, exchanging ones to tens, exchanging tens to hundreds)						
	add a three-digit number and hundreds (using numbers up to 1000)							
	(no bridging the hundreds, bridging the hundreds)	45						
	Subtraction Mentally (Including Jottings)	Subtraction Written						
	subtract a three-digit number and ones	subtract numbers with up to three digits, using formal written methods of						
	subtract a three-digit number and tens	columnar subtraction						
	9	(No exchanging, exchanging tens to ones, exchanging hundreds to tens)						
	subtract a three-digit number and hundreds	Apparatus should						
	(no bridging the hundreds, bridging the hundreds)	$\begin{array}{cccccccccccccccccccccccccccccccccccc$						

Multiplication Mentally (Including Jottings)						Multiplication Written				
<ul> <li>recall and use multiplication facts for the 3, 4 and 8 multiplication tables</li> <li>write and calculate mathematical statements for multiplication using the multiplication tables that they know including for two digit numbers times</li> </ul>					write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers time one-digit numbers progressing to formal written methods (No exchanging, exchanging ones to tens, exchanging tens to hundreds)					
35 x 4 $70 \times 2 = 140$ $35 \times 4$ $31 \times 4 = 12$ $5 \times 4 = 20$ $140$					2 4 <u>x 8</u> <u>1 9 2</u>	mental with jot	n should be secure on methods of multiplication ings before moving on to ndard written method			
Division Mentally (Incl	udina Jottinas)				Divis	ion Written				
recall and use division facts for the 3, 4 and 8 multiplication tables					write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers (no remainders, remainders)					
9, 12, 15, 18, 21, 24, 2	7 9 x 3	= 27	64 ÷ - <u>40</u> 24 - 24	_		= 16 10 x 4 6 x 4	Children should be secure on mental methods of division with jottings before moving on to this chunking method			
i	e mathematical statements that they know, including the strategic statements, using mental strategic strat	Itiplication facts for the 3, 4 and 8 multiple mathematical statements for multiplicates that they know, including for two-digits, using mental strategies $ \begin{array}{cccccccccccccccccccccccccccccccccc$	Itiplication facts for the 3, 4 and 8 multiplication tables are mathematical statements for multiplication using the less that they know, including for two-digit numbers times is, using mental strategies $ \begin{array}{cccccccccccccccccccccccccccccccccc$	Itiplication facts for the 3, 4 and 8 multiplication tables e mathematical statements for multiplication using the es that they know, including for two-digit numbers times is, using mental strategies  35 x 4 3 x 4 = 12 5 x 4 = 20 140  20  35 x 4 3 x 4 = 12 5 x 4 = 20 140  20  20  35 x 4 3 x 4 = 12 5 x 4 = 20 4 5 x 4 = 20 4 64 ÷ $-\frac{40}{24}$	Itiplication facts for the 3, 4 and 8 multiplication tables e mathematical statements for multiplication using the est that they know, including for two-digit numbers times is, using mental strategies  • write and calculate multiplication tables one-digit numbers properties (No exchanging, exchanging, exchanging)  120 35 x 4 3 x 4 = 12 5 x 4 = 20 140  • write and calculate multiplication tables $\frac{x}{3}$ $\frac{3}{3}$ $\frac{3}{3}$ $\frac{x}{3}$ $\frac{3}{3}$ $\frac{3}{3}$ $\frac{x}{3}$ $\frac{x}{3}$ $\frac{3}{3}$ $\frac{x}{3}$ $\frac{x}{3$	Itiplication facts for the 3, 4 and 8 multiplication tables e mathematical statements for multiplication using the set that they know, including for two-digit numbers times is, using mental strategies  120	Itiplication facts for the 3, 4 and 8 multiplication tables are mathematical statements for multiplication using the east that they know, including for two-digit numbers times is, using mental strategies  • write and calculate mathematical statements multiplication tables that they know, including one-digit numbers progressing to formal writt (No exchanging, exchanging ones to tens, exchanging)  120 35 x 4 3 x 4 = 12 12 x 10 = 120 5 x 4 = 20 140  • write and calculate mathematical statements multiplication tables that they know, including one-digit numbers progressing to formal writt (No exchanging, exchanging ones to tens, exchanging)  13 28 24 $\frac{x}{3}$			

Addition Mentally (Including Jottings)	Addition Written				
It would be helpful if children could begin to add 4 digit numbers mentally where appropriate, as Yr 3 add 3 digit and Yr 5 add 5 digit. (no bridging, bridging) (include jottings where necessary) 4 digit + 1s 4 digit + 10s 4 digit + 100s	add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate  (Exchanging ones to tens, exchanging tens to hundreds, exchanging hundreds to thousands)      2 3 4 5				
Subtraction Mentally (Including Jottings)	Subtraction Written				
It would be helpful if children could begin to subtract 4 digit numbers mentally where appropriate, as Yr 3 subtract 3 digit and Yr 5 subtract 5 digit. (no bridging, bridging) (include jottings where necessary) 4 digit - 1s 4 digit - 10s 4 digit - 100s Finding the difference between 2 larger numbers close together can be taught by counting/jumping up from the lowest. 1000 – 998 = 2	• subtract numbers with up to 4 digits using the formal written methods of columnar and subtraction where appropriate  (Exchanging tens to ones, exchanging hundreds to tens, exchanging thousands to hundreds)  31 23 15 23 45 23 45 -127 -163 -1523 -1523 -182 -0822				
Multiplication Mentally (Including Jottings)	Multiplication Written				
<ul> <li>recall multiplication facts for multiplication tables up to 12 x 12 (6,7,9,11,12)</li> </ul>	<ul> <li>Multiplication Written</li> <li>multiply two-digit and three-digit numbers by a one-digit number using formal written layout</li> <li>2 4</li> <li>× 6</li> <li>1 4 4</li> <li>2</li> </ul>				
	It would be helpful if children could begin to add 4 digit numbers mentally where appropriate, as Yr 3 add 3 digit and Yr 5 add 5 digit. (no bridging, bridging) (include jottings where necessary) 4 digit + 1s 4 digit + 10s 4 digit + 10os  It would be helpful if children could begin to subtract 4 digit numbers mentally where appropriate, as Yr 3 subtract 3 digit and Yr 5 subtract 5 digit. (no bridging, bridging) (include jottings where necessary) 4 digit - 1s 4 digit - 10s 4 digit - 10s 5 digit - 10s 7 Finding the difference between 2 larger numbers close together can be taught by counting/jumping up from the lowest. 1000 – 998 = 2    Multiplication Mentally (Including Jottings)   recall multiplication facts for multiplication tables up to 12 x 12 (6,7,9,11,12)   use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers   180				

Division Mentally (Includeing Jottings)	Division Written
<ul> <li>recall division facts for multiplication tables up to 12 x 12</li> </ul>	write and calculate mathematical statements for division using the
use place value, known and derived facts to divide mentally	multiplication tables that they know including a 3 digit number divided by a 2 digit number. (no remainders, remainders)
240 ÷ 6 =	846 ÷ 6 =
$24 \div 6 = 4$ $240 \div 6 = 40$	- 600 100 x 6 Answer to each subtraction can be
divide by 1	- 60 10 x 6 added in to the
multiply together three numbers	- 60 10 x 6 calculation if required. See Yr3.
	- 60 10 x 6
	- 6 1 x 6

	Addition Mentally (Including Jottings)	Addition Written				
Year 5	<ul> <li>add numbers mentally with increasingly large numbers</li> <li>10573 + 3200 = 13773</li> <li>add numbers to 2 decimal places</li> <li>9.58 + 3.08 = 12.66</li> </ul>	add whole numbers with more than 4 digits, including using efficient written methods (columnar addition)  44388 + 5896 - 50284 - 1111  add numbers to 3 decimal places 32.148 + 9.738 - 41.886 1 1 1				
	Subtraction Mentally (Including Jottings)	Subtraction Written				
	<ul> <li>subtract numbers mentally with increasingly large numbers</li> <li>64501 – 4300 = 60201</li> <li>subtract numbers to 2 decimal place</li> <li>7.47 – 3.15 = 4.32</li> </ul>	• subtract whole numbers with more than 4 digits, including using efficient written methods (columnar subtraction)   • 13 1 • 14 6 7 - 26 8 4 - 37 8 3  • subtract numbers to 3 decimal places  21 1 • 12 148 - 9.738 - 22.410				

Multiplication Me	ntally (Including Jottings)	Multiplication Written
45 x 6 =  45 x 2 = 90, 90 x 3 (9x3x10)  = 270  38 x 15 =  38 x 10= 380 + 190 (1/2 of 380)  = 570  • solve problems involving multiple decomposing them into their fact 350 x 25  = 7 x 5 x 10 x 5 x 5	ication where larger numbers are used by etors  se involving decimals by 10, 100 and 1000  When multiplying by 10 and multiples of 10, the decimal point remains fixed and the number moves 1 place to the left for each multiple of 10	<ul> <li>multiply numbers up to 4 digits by a one-digit number using an efficient written method  4346  x 8 34768 234</li> <li>multiply numbers up to 4 digits by a two-digit number using long multiplication 4115 x 25 4115  x 25 20575  1 82300 102875 recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³) 2²=2 x 2 = 4 3³=3 x 3 x 3 = 27</li> </ul>
divide numbers mentally drawin	g upon known facts ers involving decimals by 10, 100 and 1000  When dividing by 10 and multiples of 10, the decimal point remains fixed and the number moves 1 place to the right for each multiple of 10	Division Written  • divide numbers up to 4 digits by a one-digit number using the efficient written method of short division and interpret remainders appropriately for the context  72 ÷ 5  14 r 2  5

	Addition Mentally (Including Jottings)	Addition Written				
Year 6	add numbers mentally with increasingly large numbers	add whole numbers with more than 4 digits, including using efficient written methods (columnar addition) (See Year 5 for examples)				
	Subtraction Mentally (Including Jottings)	Subtraction Written				
	subtract numbers mentally with increasingly large numbers	subtract whole numbers with more than 4 digits, including using efficient written methods (columnar subtraction) (See Year 5 for examples)				
	Multiplication Mentally (Including Jottings)	Multiplication Written				
		-				
	perform mental calculations, including with mixed operations and large numbers	multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication				

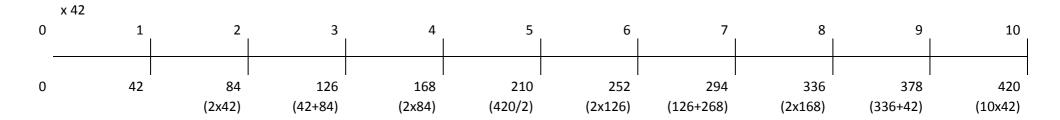
Division Mentally (Including Jottings)	Division Written				
perform mental calculations, including with mixed operations and large numbers	divide numbers up to 4 digits by a two-digit whole number using the efficient written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate to the context				
	2666 ÷ 42 Remainder $ \begin{array}{cccccccccccccccccccccccccccccccccc$				
	Fraction  63 20/42  42)2666  252 146 126 20  See Appendix 1 for Bar and Double Number Line representations				
	Decimal 63.47 42)266600 -252 146 -126 200 -168 320 -294 26				

## Tools to aid multiplication in division calculations

#### **Bar Method**

	x 42									
0	1	2	3	4	5	6	7	8	9	10
0	42	84	126	168	210	252	294	336	378	420
		(2x42)	(42+84)	(2x84)	(420/2)	(2x126)	(126+268)	(2x168)	(336+42)	(10x42)

## **Double Number Line Method**



This method of multiplication uses know or easily computable facts, by doubling and halving or addition, to construct the data needed to carry out the calculation. There is no need to calculate all values from 1 times to 10 times, only those needed.